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Natural convection mass transfer at conical surfaces

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Rates of natural convection mass transfer were determined at vertical cones by measuring the limiting current of the cathodic deposition of copper from acidified copper sulphate solution. Variables studied were cone position (upward or downward apex), apex angle of the cone and copper sulphate concentration. For vertical cones with upward pointing apex the data were represented in the range $4.9 \times 10^{10} < (Sc\ Gr) < 9 \times 10^{11}$ by the equation:

$$Sh = 0.128\ (Sc\ Gr)^{0.33}$$

For vertical cones with downward pointing apex the data fit the equation:

$$Sh = 0.877\ (Sc\ Gr)^{0.25}$$

Nomenclature

C	Cu^{2+} concentration.
D	diffusivity of Cu^{2+} ions.
F	Faraday number (96 500 C mol^{-1}).
g	gravitational acceleration.
Gr	Grashof number ($gL^3\Delta\varrho/\nu^2\varrho_i$).
H^0, H	a reference cone height and cone height respectively [10].
I_L	limiting current density.
K	mass transfer coefficient.
L	slant height of the cone.
Sc	Schmidt number ($\mu/\varrho D$).
Sh	Sherwood number (KL/D).
Ra	Rayleigh number ($Sc\ Gr$).
Z	number of electrons involved in the reaction.
ν	kinematic viscosity of the fluid.
μ	dynamic viscosity of the fluid.
ϱ	density of the fluid.
$\Delta\varrho$	density difference between bulk and interface.
ϱ_i	density of the fluid at the interface.
θ	apex half angle [10].

1. Introduction

Although much work has been done on natural convection heat and mass transfer at different transfer geometries [1] such as vertical and horizontal plates, vertical and horizontal cylinders, vertical and horizontal tubes, spheres and inclined surfaces, little has been done on the natural convection heat and mass transfer behaviour of conical surfaces despite the frequent occurrence of this geometry in practice either alone or as a part of more complex shapes. Stewart [2] made a theoretical analysis of free convection transfer at a vertical cone (apex downward) with insulated base, his analysis led to the equation:

$$Sh = 0.64\ (Gr\ Sc)^{0.25} \quad (1)$$

Weber *et al.* [3] studied the rate of free convection mass transfer at cones with active bases by measuring the limiting current of the cathodic deposition of copper from acidified copper sulphate solution. Variables studied were apex angle, cone orientation and copper sulphate concentrtion. The authors correlated their data by the equation:

$$Sh = Sh_0 + 0.665\ (Sc\ Gr)^{0.248} \quad (2)$$

In calculating Sh and Gr, the authors used a characteristic length L given by:

$$L = A/P \quad (3)$$

where A is the surface area and P is the perimeter of the cone projected on a horizontal plane while Stewart used the slant height of the cone as a characteristic length.

Patrick and Wragg [4] studied the natural convection mass transfer behaviour of upward and downward pointing cones with isolated base within the range $2 \times 10^8 < Ra < 10^{11}$. The authors correlated their data for downpointing cones with the equation

$$Sh_L = 0.87\ Ra^{0.25} \quad (4)$$

Slant height was used as a characteristic length and $g \cos\theta$ was used in calculating Ra. For the upward pointing cones the data were correlated with the equation.

$$Sh_R = 0.034\left[Sc\ Gr_R \sqrt{\frac{H^0}{H}}\right]^{0.4} \quad (5)$$

Base diameter (R) was used as a characteristic length. The objective of the present work is to provide more data under different conditions with the hope of confirming and/or improving previous correlations. Mass transfer at conical surfaces is important in determin-

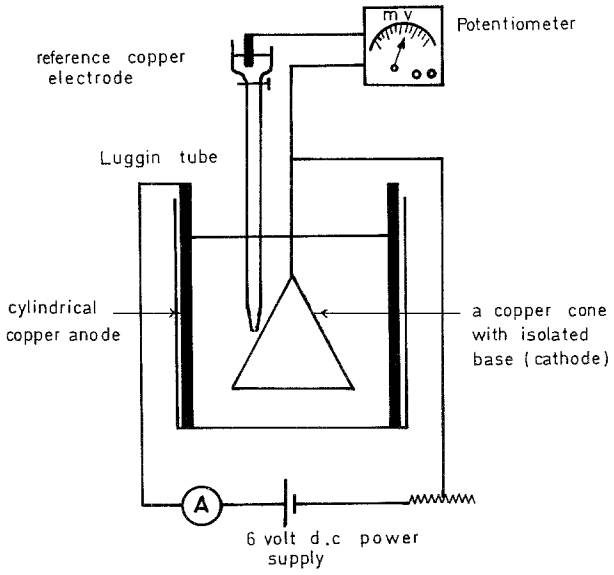

Fig. 1. Cell and electrical circuit.

ing the kinetics of diffusion controlled reactions which may take place at a conical surface in unstirred or slightly stirred solutions, e.g. electropolishing, electroplating and corrosion. Besides, the present work may be of value to free convection heat transfer at conical surfaces by virtue of the analogy between heat and mass transfer.

2. Experimental details

Fig. 1 shows the cell and circuit used. The cell was a cylindrical plexiglass container 15 cm in diameter and 20 cm high. The anode was a cylindrical copper sheet of 14.5 cm diameter and 20 cm height. The cathode was a cone made of copper held in position by an insulated 2 mm diameter copper wire brazed either to the apex or the base for cones with upward and downward pointing apexes respectively. The wire also acted as a current feeder. The base diameter of the cones ranged from 5.2 to 10.6 cm and slant height ranged from 9.5 to 10.5 cm, apex angle ranged from 32 to 58° (Table 1). The cone was positioned in the cell with its apex in the centre of the container. The circuit consisted of a 6 V d.c. power supply with a voltage regulator connected in series with a multirange ammeter and the cell. Polarization curves from which the limiting current was determined were constructed by increasing the cell current stepwise and measuring the corresponding steady state cathode potential. The latter was measured against a reference electrode made of a copper wire placed in the cup of a Luggin tube by means of a high impedence voltmeter. The Luggin tube was filled with a solution identical with the cell solution with its tip placed 0.5–1 m from the cone surface. Solutions used were prepared from AR copper sulphate and sulphuric acid. Copper sulphate concentration ranged from 0.05 to 0.46 M; 1.5 M H_2SO_4 was used as a supporting electrolyte. Before each run, the cathode surface was treated as mentioned elsewhere [5]. The cone base was isolated with epoxy. Each experiment was repeated twice. The temperature was 22 ± 1°C.

Table 1. Dimensions of the cones used

Base diameter (cm)	*Slant height* (cm)	*Apex angle* (°)
10.6	10.5	58
9.15	9.6	56
8	9.75	48
7	9.7	42
6.2	9.9	36
5.2	9.5	32

3. Results and discussion

Polarization curves with a well-defined limiting current plateau were obtained under different conditions; from these polarization curves the limiting current was determined and used in calculating the mass transfer coefficient according to the equation:

$$K = \frac{I_L}{ZFC} \tag{6}$$

An overall mass transfer correlation was obtained in terms of the dimensionless groups *Sh*, *Sc* and *Gr* usually used in correlating natural convection mass transfer data; the physical properties (ϱ, μ, D) of the solutions used to calculate these dimensionless groups were obtained from the literature [5, 6]. Following Stewart [2], the slant height of the cone was used as a characteristic length. Fig. 2 shows that data for cones with upward pointing apex can be represented in the range $4.9 \times 10^{10} < (Sc\ Gr) < 9 \times 10^{11}$ by the equation

$$Sh = 0.128\ (Sc\ Gr)^{0.33} \tag{7}$$

With an average deviation of ±5.3% while at a vertical cone with downward pointing apex, Fig. 3 shows that the data can be represented in the range $4.9 \times 10^{10} < (Sc\ Gr) < 9 \times 10^{11}$ by

$$Sh = 0.877\ (Sc\ Gr)^{0.25} \tag{8}$$

with an average deviation of ±5.5%. The (*Sc Gr*) exponents 0.33 and 0.25 shown in equation 7 and 8 reveal a turbulent flow and laminar flow mechanism at the cone with upward apex and the cone with downward apex respectively. The turbulent flow mechanism at vertical cones with upward apex is consistent with the finding of different authors [7–11] who measured the rate of natural convection mass transfer at upward facing inclined plate cathode where eddy formation and boundary layer separation take place especially at high *Sc Gr*, as used here. The laminar flow mechanism at vertical cones with a downward apex is also consistent with the results of different authors who measured mass transfer at downward facing inclined plates where laminar flow is likely to take place [7–11]. It is difficult to compare equations 7 and 8 with equation 2 which was obtained by Weber *et al.* [3] because (i) Weber *et al.* [3] used cones with active base; (ii) the

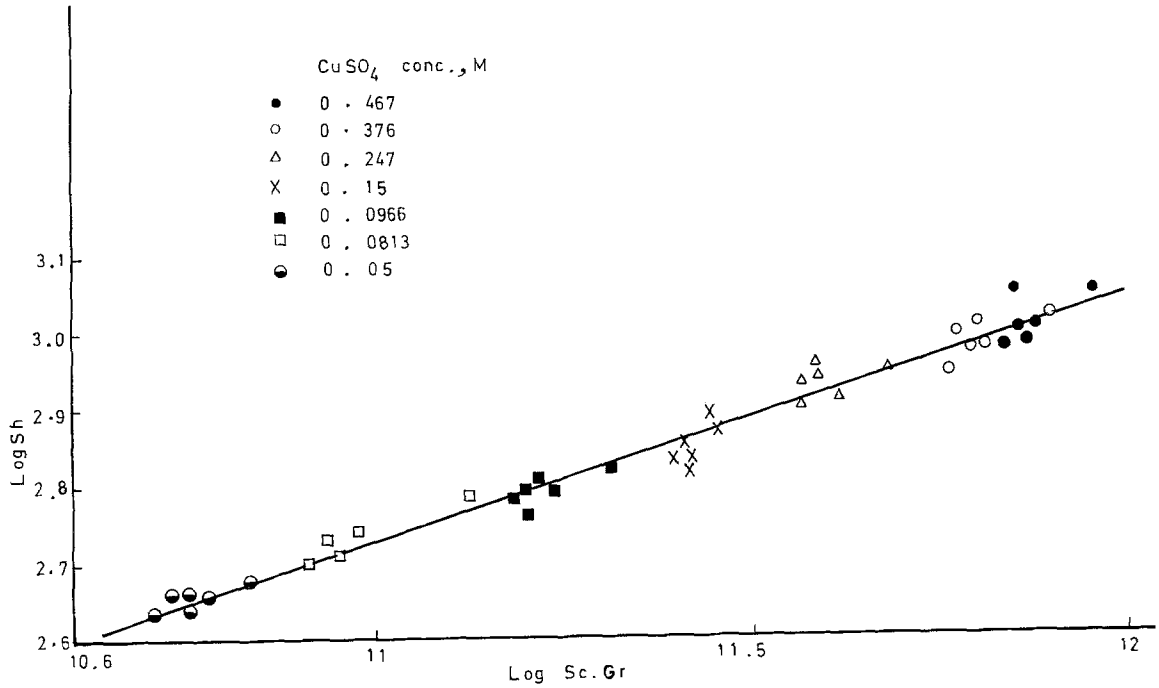

Fig. 2. Overall mass transfer correlation for cones with upward apex.

data used to obtain equation 2 represent vertical cones with apex upward, vertical cones with apex downward and horizontal cones; besides, Weber *et al.* used a different characteristic length as defined by equation 3. Although equation 2 represents the data of Weber *et al.* quite well it does not reveal the mechanism of mass transfer at the conical surface because of the interaction between mass transfer at the base and at the conical surface. For a vertical cone with upward apex, the flow at the conical surface may be turbulent while at the downward facing base, convection is laminar. While in the case of a vertical cone with downward apex, flow at the upward facing base may be laminar or turbulent, and flow at the conical surface may be laminar. Equation 8 which correlates mass transfer data at a vertical cone with downward apex agrees fairly well with the theoretical equation of Stewart (equation 2).

The date of Patrick and Wragg [4] for downpointing cones are in a good agreement with equation 8, the average deviation is $\pm 6\%$. However for upward facing cones the present data deviate from the equation obtained by Patrick and Wragg, equation 5, by an average value of $\pm 21.3\%$.

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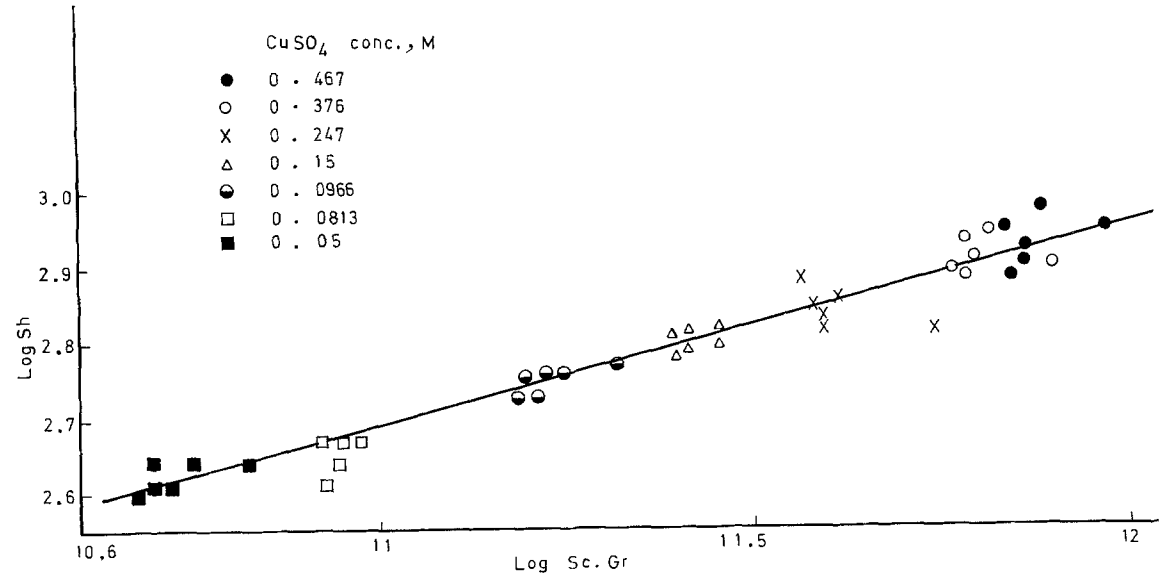

Fig. 3. Overall mass transfer correlation for cones with downward apex.

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